Comment on quant-ph/9509008 by Kumar and Khare

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One can find some comments related to the isospectral issue.

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Recently, Kumar and Khare [1] constructed in a general manner coherent states for strictly isospectral Hamiltonians H_λ, where λ is the isospectral family parameter, by using a unitary transformation relating H_λ to the original Hamiltonian $H_{|\lambda|\rightarrow\infty}$. This is different from old proposals regarding the isospectrality issue [2,3], and appears as the natural construction for the discrete part of the spectrum. This is why, Kumar and Khare commented in the negative on the old results on this problem. However, without questioning the good merits of their work, I have found the remarks they did relative to [2] as too negative, though perhaps some statements in [4] may be judged as incorrect. Therefore, I decided to write this comment.

Mielnik's "annihilation" and "creation" operators [2] are in fact only factorization operators, since they do not commute to the identity. Thus, they do not reduce to the common pair of conjugated oscillator operators $(a, a^\dagger)$ in the isospectral infinite limit but merely to nonlinear product type operators $(a^\dagger a^2,\ (a^\dagger)^2 a)$. They connect the discrete part of the spectrum to the continuum part and vice versa, and in fact this is the reason why the old isospectral construction is directly related to inverse scattering methods [3]. In group theory terminology, the factorization operators mix Fock representations (bound states) and non-Fock ones (continuous spectrum). Such a mixing leads, in the isospectral infinite limit, to the aforementioned (cubic) nonlinear operators. The true nature of such operators is not yet very clear, and depend on the particular case under consideration. The limit, itself, suggests a possible connection by means of the isospectral parameter between the linear Schrödinger equation and the cubic nonlinear one. I mention that recently, Andrianov *et al* [5] related the bound and continuous part of a spectrum by means of local q-deformed oscillators. The relation between the isospectral parameter and the q-parameter (q-self-similarity) is an interesting open issue. Solitonic operators, such as Mielnik ones, may be quite useful in more concrete applications, as they reflect the connection between the bound and unbound spectra, which cannot be avoided.

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